Lesson 5.1 • Hot and Cold — CBL 2

In this demonstration you will use two temperature probes to collect data from two different cups of water as you add ice cubes to one and hot water to the other one.

Materials Needed: CBL 2, two temperature probes, containers of hot and cold water, two cups, a coffee measure, ice cubes, a stopwatch or watch with a second hand

Experiment

Step 1 Fill one cup half-full with hot water. Fill the second cup half-full with cold water. You will need room to add ice cubes to the hot water and hot water to the cold water.

Step 2 Create a set of axes with time in seconds on the x-axis and temperature in degrees Celsius on the y-axis. Draw a graph of your prediction for how the temperature will change in each cup as you add ice cubes to the hot water and hot water to the cold water over a 4-minute time period.

Step 3 Plug one temperature probe into Channel 1 and the other one into Channel 2 of the CBL 2 and connect your calculator using the link cable.

Step 4 Start the EasyData application on the calculator. The CBL 2 will automatically detect the two temperature probes.

Step 5 Press Setup (WINDOW). Choose Time Graph and press ENTER. Press Edit (ZOOM). Enter 1 for the time between samples and press Next, then enter 240 for the total number of samples and press Next. Press OK and then press Start.

Step 6 Start the stop watch. Add an ice cube to the cup of hot water. Add one coffee measure of hot water from the container to the cup of cold water. Repeat this process every 30 seconds. When data collection is done, press Main, press Quit, and press OK.

Investigate

1. Press GRAPH and sketch the resulting graph.
2. Find equations for each data set that fit reasonably well. Record your equations and briefly describe the method you used to find each equation.
3. Find an approximate point of intersection. What is the real-world meaning of the intersection point?
4. Suppose you repeated this activity, but this time you added two ice cubes to the hot water and you added two coffee measures of hot water to the cup of cold water. How would your graphs, equations, and intersection point change?
5. Suppose you repeated this activity, but this time instead of adding ice cubes, you added ice-cold water to the cup of hot water. How would your graph change?

Lesson 5.1 • Solving Systems of Equations

The total tuition for students at University College and State College consists of student fees plus costs per credit. Some classes have different credit values. The table shows the total tuition for programs with different numbers of credits at each college.

Credits	Total Tuition at University College ($)	Total Tuition at State College ($)
1	55	47
3	115	111
6	205	207
9	295	303
10	325	335
12	385	399

In this demonstration you will use Sketchpad to graph a system of equations to model the cost of each college.

Sketch

Step 1 Choose **New Sketch** from the File menu.

Step 2 Choose **Rectangular Grid** from the Graph | Grid Form menu.

Investigate

1. For a graph of (*Credits, Tuition*), what are appropriate minimum and maximum values for the x-axis? The y-axis?
2. Write a system of equations that represents the relationship between credits and total tuition for each college.

Sketch

Step 3 Because you don't need negative values for the data, drag the origin (point (0, 0)) to the lower-left corner of your sketch.

Step 4 Drag the *unit* point on the x-axis (the point that lies at (1, 0)) to change the scale so that all the data points will appear on your graph. Do the same for the y-axis.

Step 5 Choose **Plot Points** from the Graph menu.

Step 6 With Rectangular selected, enter 1 and 55 to plot the first point from the table. Click Plot. Plot (9, 295) and then click Done.

Step 7 Select both of the points you just plotted. Choose **Line** from the Construct menu.

(continued)

Step 8 With the line selected, choose the color blue from the Display | Color menu. Select the **Label** tool (the A on the tool bar) and click on the line. Double-click on the label that appears, and change the label to University College. Drag the label to the right of the line.

Step 9 Repeat Steps 5 to 8 for the points (1, 47) and (9, 303). Select a different color for this line, and label it "State College."

Step 10 Select both lines. Choose **Equations** from the Measure menu to check your answer to Question 2.

Investigate

3. Where do the lines appear to meet?

4. What is the real-world meaning of this point?

Sketch

Step 11 Select both lines. Choose **Intersection** from the Construct menu.

Step 12 Select the intersection. Choose **Coordinates** from the Measure menu to check your answer to Question 3.

Investigate

5. When is it cheaper to attend University College? Explain how you can tell from the graph.

6. When is it cheaper to attend State College?

Lesson 5.2 • Olympic Times — Fathom

This demonstration shows how to find Q-line models and how to graphically approximate a point of intersection of two lines.

Experiment

Step 1 Open the document **Breaststroke.ftm,** which gives the winning times for Olympic gold medalists in the 100-meter breaststroke.

Step 2 On a new graph, plot the women's times as (*Year, Wtime*).

Step 3 Find the Q-points by plotting values for Q1(*Year*) and Q3(*Year*) and plotting functions at Q1(*WTime*) and Q3(*Wtime*).

Investigate

1. Use the Q-points to find a line of fit for the women's times. Check your result by plotting the equation. (If you are getting the #Units Incompatible# message, add units to each part of your equation.)
2. On a new graph, repeat Steps 2 and 3 for the men's times. What are the Q-points for the men's times?
3. Find an equation for the men's Q-line. Plot the function to check. (Again, include the units.)

Experiment

Step 4 Graph the equation for the men's times on the graph of the women's times.

Step 5 Drag the vertical and horizontal axes to change the scales until you can see both lines and the point where they intersect.

Investigate

4. If you point the cursor at one of the lines, a red dot will appear. Its coordinates will show in the bottom-left corner of the window. Move the dot to find the approximate coordinates of the point of intersection.
5. What do these coordinates mean in terms of times and years? Does this answer seem reasonable? Explain.
6. Solve a system of equations by substitution to get this information exactly.

Lesson 5.6 • Graphing Inequalities Sketchpad

In this demonstration you will discover the properties of graphs of inequalities in two variables.

Sketch

Step 1 Open the document **Inequalities.gsp** to the page One Inequality.

Step 2 Press the *Show Graph* button.

Step 3 Press the $< >$ button several times while watching the changes in the equation and graph.

Investigate

1. Describe how the graph changes when the inequality symbol changes from $<$ to $\leq$ and from $>$ to $\geq$.

2. What do you think the shaded region represents?

Sketch

Step 4 Press the $< >$ button to show a shaded region. Press the *Show Point* button.

Step 5 Drag point F to any part of the shaded region in your sketch.

Investigate

3. Substitute the coordinates of F into the inequality. Is this a true or false statement?

4. Repeat using another point in the shaded region. What does the shaded region represent?

Sketch

Step 6 Select parameter a. Use the "+" or "−" keys to change the value of a to -2.

Step 7 Repeat Step 6 to adjust b and c and press the $< >$ button to obtain the inequality $-2x - 5y < 6$.

Investigate

5. Is (2, 3) a solution to this inequality? Explain how you can tell from the graph.

6. Is $(-3, 0)$ a solution? $(0, -4)$? Explain using both the inequality and the graph.

(continued)

Sketch

Step 8 Go to the page System of Inequalities.

Step 9 Press the *Show Graph* button for each equation.

Step 10 Repeat Step 6 to adjust the a, b, and c values to graph the equations $5x - 2y = 1$ and $6x - 8y = 4$.

Step 11 Select both lines. Choose **Intersection** from the Construct menu.

Step 12 Select the intersection. Choose **Coordinates** from the Measure menu.

Investigate

7. What does this point represent?

Sketch

Step 13 Press the $<$ $>$ buttons to graph the system of inequalities $5x - 2y < 1$ and $6x - 8y > 4$.

Investigate

8. Choose three points, one from each shaded region. Substitute each point into both inequalities. Describe your results.

9. What does the overlapping shaded region represent?

10. Is $(0, -2)$ a solution to this system of inequalities? Explain.

Lesson 5.7 • Linear Programming Sketchpad

In this demonstration you will use systems of inequalities to explore the following exercise:

> On Kid's Night, every adult admitted into a restaurant must be escorted by at least one child. The restaurant has a maximum seating capacity of 75 people. On average, the restaurant makes a profit of $5 per child and $8 per adult. How many adults and children need to eat at the restaurant to maximize the restaurant's profit? What is that profit?

Investigate

1. Define variables and write a system of inequalities that models this situation.
2. Graph the inequalities.

Sketch

Step 1 Open the document **LinearProgramming.gsp** to the page System.

Step 2 Press the *Inequality 1* button.

Investigate

3. What constraint does this inequality represent?
4. What do the variables represent? Name three points that make the inequality true and describe what they represent.

Sketch

Step 3 Press the *Inequality 2* button.

Investigate

5. What constraint does this inequality represent?
6. What does the overlapping region represent?
7. Write two other constraints needed for this exercise.

Sketch

Step 4 Go to the page Profit in the same document.

Step 5 Press the *Show Point* button.

Step 6 Drag point (C, A) around the sketch.

(continued)

Investigate

8. Describe two ways to determine whether point (*C*, *A*) satisfies the constraints.

9. According to the rules for Kid's Night, if there are five children in the restaurant, how many adults can there be? *Hint:* There is more than one answer.

10. Write an equation that will give the profit.

11. What will the restaurant's profit be if there are five children and three adults?

Sketch

Step 7 Press the *5 Children* button to check your answer to Question 9.

Step 8 Press the *Show Profit* button.

Step 9 Drag point (*C*, *A*) to the point (5, 3) to check your solution to Question 11.

Step 10 Drag point (*C*, *A*) around the solution region to find the maximum profit.

Investigate

12. What is the maximum profit possible, given the constraints?

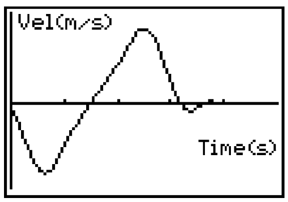
13. How many children and adults must attend to achieve the maximum profit?

paper towels or a coffee can. The sample data were collected using a 1.25 m ramp with a height of 15 cm. Before you begin the activity, you might want to discuss the difference between velocity and speed with your students. Velocity has direction, so it can be negative or positive. Speed has no direction; therefore, it is never negative. If you are using a TI-73 or TI-83, use the DataMate program.

Lesson guide

In this demonstration students find a linear model in point-slope form for the velocity of a tube as it rolls down a ramp. Point-slope form works well because there will probably be some initial “noise” in the data.

Investigate



1. A sample graph is given. Make sure that students label the units for each axis. There will probably be some “noise” in the data because the tube will not start rolling at exactly 0 s and may finish rolling before 5 s. In the sample graph, the portion from about 0.6 s to 2.5 s can be modeled with a line.
2. Using the sample data, (0.97, −0.34) and (1.99, 0.28) are the points chosen. The equation $y = -0.34 + 0.61(x - 0.97)$ fits the uphill part of the sample data well.
3. The velocity is positive between about 1.5 s and 2.5 s. The velocity is positive when the tube is rolling away from the CBR.
4. The velocity is negative between about 0.6 s and 1.5 s. The velocity is negative when the tube is rolling toward the CBR.
5. The velocity is zero at about 1.5 s. The velocity is zero when the tube changes direction and starts to roll back down the ramp.

LESSON 4.6 • More on Modeling

Required document

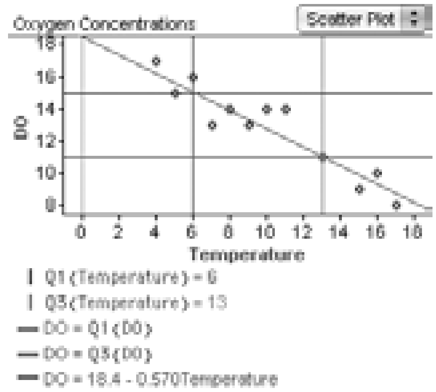
Oxygen.ftm

Lesson guide

This demonstration replaces the Example of Lesson 4.6, drawing a line through Q-points. In Fathom, students see how to plot quartiles Q1 and Q3 on both axes.

Investigate

1. Equations will look very much like the actual equation, approximately $DO = 18.4 - 0.57\,Temperature$.



2. The Q-points are (6, 15) and (13, 11). The equation is $DO = 11 - \frac{4}{7}(x - 13)$, or approximately $DO = 18.4 - 0.57\,Temperature$. *Note:* click on the graph to make the Graph menu available.
3. About half the residuals are above 0 and half are below 0, so it’s a pretty good representation.
4. About 25°C

CHAPTER 5

LESSON 5.1 • Hot and Cold

Required materials

Calculator with DATAMATE application, CBL 2, two temperature probes, container with hot water, container with cold water, two large empty cups, seven to ten ice cubes, a coffee measure, a stop watch or a watch with a second hand

Activity notes

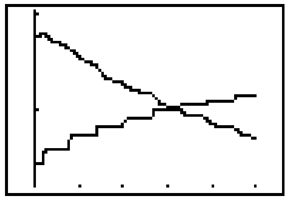
This demonstration is a practical application of a system of equations. The data will be roughly linear. Hot tap water and cold tap water work fine. Make sure that there is room in each cup to add ice cubes or hot water. You can stir each time you add an ice cube or hot water to get better data. If you have a TI-73 or TI-83, use the DataMate program.

Lesson guide

This lesson further develops the skills of writing equations to fit data that students learned in Chapters 3 and 4. It continues their study of linear equations by introducing a system of two equations with two unknowns.

INVESTIGATE

1. When the CBL 2 is done collecting data, students should press **Main** and press **Quit**. When they press $\boxed{\text{GRAPH}}$, they should see a scatter plot of the temperature values vs time. A sample graph is given. Make sure that students label the units for each axis on their sketch.



2. The equations $y = -0.107x + 43.74$ and $y = 0.072x + 13.57$ fit the sample data reasonably well. Encourage students to use one of the methods discussed in Chapter 4 for finding a linear model.
3. (168.54, 25.7) is the approximate intersection point of the sample graph. The intersection point is the time at which the two cups of water have the same temperature. The sample graph shows that both cups of water had a temperature of about 25.7°C after about 169 s.
4. Both graphs should be steeper because the hot water will cool at a faster rate and the cold water will warm up at a faster rate. The intersection point should be earlier.
5. Students may have noticed that the graph of the cold water has "jumps" where the temperature changes abruptly, while the graph of the hot water changes more smoothly. Adding ice-cold water, rather than ice cubes, will cause the hot water graph to show similar jumps in temperature; adding ice cubes makes the temperature change less abruptly.

LESSON 5.1 • Solving Systems of Equations

REQUIRED DOCUMENT

None

LESSON GUIDE

In this demonstration students will use the graphing features of Sketchpad to investigate Exercise 8. They will change the scale of each axis, plot points, construct a line, and find the point of intersection.

INVESTIGATE

1. Answers will vary. Possible answer: x minimum: 0, maximum: 12; y minimum: 0, maximum 400
2. $y = 25 + 30x$; $y = 15 + 32x$
3. (5, 175)
4. At both colleges it costs $175 to take 5 credits.
5. It is cheaper to attend University College if you are taking more than 5 credits. This is shown on the graph when the University College line is lower than the State College line.
6. It is cheaper to attend State College if you are taking less than 5 credits.

LESSON 5.2 • Olympic Times

REQUIRED DOCUMENT

Breaststroke.ftm

LESSON GUIDE

This demonstration can replace Exercise 12 of Lesson 5.2. Students find the Q-points by graphing the Q1 and Q3 values, find the equation and graph the Q-line, and use the graph of the system to make a prediction. In Fathom, students learn to add units to their equation when graphing attributes with units.

INVESTIGATE

1. The Q-points are at (1976, 71.16) and (1996, 67.73). The equation is $WTime = 71.16 - 0.1715(Year - 1976)$, or $WTime = 67.73 - 0.1715(Year - 1996)$. To graph correctly with units, the equation should be entered as WTime = 71.16 s – 0.1715 s/yr(Year – 1976) yr.
2. The Q-points are (1976, 63.44) and (1996, 60.60). The first of these is not at a data point.
3. $MTime = 63.44 - 0.142(Year - 1976)$
4. The coordinates will be near (2240, 26).
5. In about 2240 (more than 200 years from now), both men and women will swim this race in approximately 26 seconds. The model may be a good fit for the data, but extrapolating that far into the future produces unlikely predictions.
6. The actual solution is approximately (2238, 26.2).

LESSON 5.6 • Graphing Inequalities

REQUIRED DOCUMENT

Inequalities.gsp

LESSON GUIDE

Students will discover the properties of graphs of two-variable inequalities and systems of inequalities. No prior experience with graphing inequalities is necessary for this demonstration. If you don't want to introduce systems of inequalities, you can stop after Question 6.

INVESTIGATE

1. $<$ and $>$ have dashed lines and $\geq$ and $\leq$ have solid lines.
2. Answers will vary.
3. True
4. The shaded region represents points that make the inequality true.
5. Yes. It is in the shaded region.

6. $(-3, 0)$ is not a solution because it makes the inequality false and lies on the dashed line in the graph. $(0, -4)$ is not a solution because it makes the inequality false and it does not lie in the shaded region.

7. The intersection point is the solution to the system of equations. The point $(0, -0.5)$ makes each equation a true statement.

8. Answers will vary. A point from the blue region will make the first inequality true and the second inequality false. A point from the yellow region will make the first inequality false and the second inequality true. A point in the overlapping green region will satisfy both inequalities.

9. The overlapping region represents the points that make both inequalities true.

10. $(0, -2)$ is not a solution to this system because it does not lie in the overlapping region and it only makes the second inequality true.

LESSON 5.7 • Linear Programming

REQUIRED DOCUMENT

LinearProgramming.gsp

LESSON GUIDE

This demonstration can replace Exercise 7 of Lesson 5.7. It extends this exercise by using linear programming techniques to maximize the profit.

INVESTIGATE

1. Answers will vary. Sample answer: C is the number of children; A is the number of adults; $C + A \leq 75$; $A \leq C$; $C \geq 0$; $A \geq 0$

2. Graphs will vary depending on student inequalities.

3. $C + A \leq 75$; the restaurant can hold up to 75 people.

4. C is the number of children; A is the number of adults. The points should lie on the line or in the shaded region; they represent numbers of children and adults that together are less than or equal to 75.

5. $A \leq C$; every adult must be accompanied by a child, so the number of adults is less than or equal to the number of children.

6. The overlapping region shows the combinations of numbers of children and numbers of adults that make both inequalities true.

7. Students may need help determining these inequalities; $C \geq 0$; $A \geq 0$; note also that the values for C and A must be integers.

8. If (C, A) lies on the triangle or inside the shaded triangular region, then it satisfies the constraints. Substitute the coordinates into each inequality; if (C, A) makes each inequality true, then it satisfies the constraints.

9. If there are five children, there can be from zero to five adults.

10. Profit $= 5C + 8A$

11. \$49

12. \$486

13. 38 children and 37 adults

CHAPTER 6

LESSON 6.2 • Exponential Equations

LESSON GUIDE

This demonstration can replace Example B of Lesson 6.2. It shows how values of a and b in $y = ab^x$ correspond to the starting and rate values in exponential growth generated recursively. In Fathom, sliders and *caseindex* are introduced, and students learn how to add a new attribute to the left of an existing attribute in a table.

INVESTIGATE

1. One formula is $prev(Balance + 0.05Balance, 200)$. Students might use equivalent forms of $Balance + 0.05Balance$, such as $Balance(1 + 0.05)$.

2. $a = 200$, $b = 1.05$. *Note:* Fathom will not recognize ab as meaning $a \cdot b$. Be sure to enter the multiplication symbol.

3. $Balance = 200 \cdot 1.05^{Year}$

4. Students can use tracing (with a red dot) to see that the value will be \$400 in a little over 14 years.

5. Sliding the value of b shows that an interest rate of about 7% will double the account in 10 years. Students might benefit from plotting a horizontal line at $Balance = 400$ and a vertical line at $Year = 10$.

6. The starting amount appears as a, and the base b is the interest rate plus 1.

LESSON 6.7 • Fitting Exponential Models to Data

OPTIONAL DOCUMENT

Decay.ftm

LESSON GUIDE

This demonstration can supplement or replace the Radioactive Decay Investigation. It builds on the ideas in the demonstration of Lesson 6.2, but it models real data